Problematizing a Research and Development Agenda

Alan Schoenfeld

ACME 3 Opening Conference

My goal for today and tomorrow (CCME3 Conference) is to explore this issue:

If you had 5 things to focus on in order to build classrooms that produce students who are powerful thinkers, what would they be?

My answer will be the "Teaching for Robust Understanding" (TRU) Framework.

Tomorrow…

I will go into detail into the framework, tools we have built, and our attempts to build supportive professional cultures. I will raise look for points of similarity and difference with what I know of Chinese pedagogical culture.

Today…

I will start by framing the big questions, and show you how the framework evolved. I will raise a number of issues related to its origins, including how general a framework built with Western cultural assumptions might be.

My Long-Term Goal:

Building Classrooms that produce students who are Powerful Thinkers

This has been a 45-year project!

I began with problem solving:

From 1975 to 1985

I developed a theory of proficiency in problem solving indicating that the following determine success or failure:

- The knowledge base
- Problem solving strategies
- Monitoring and self-regulation
- Belief systems.

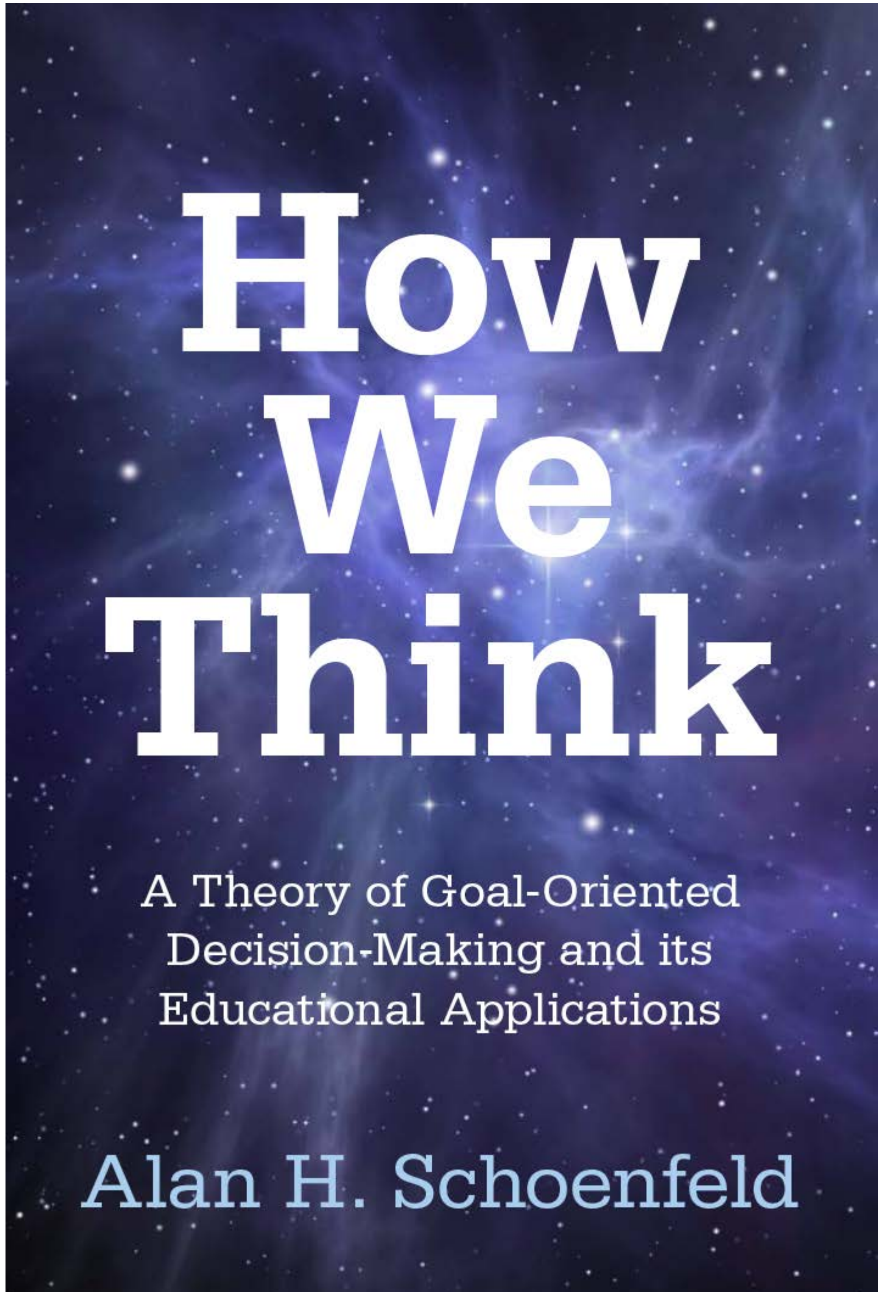
You may recognize this…

MATHEMATICAL PROBLEM SOLVING

ALAN H. SCHOENFELD

But that's about individuals and learning. What about understanding teaching?

It took another 20 years to understand teaching and, more generally, decision-making.


How We Think
A Theory of Goal-Oriented Decision-Making and its Educational Applications
Alan H. Schoenfeld

But that just focuses on one (essential) member of the classroom.

The question is, how do we focus effectively on the environment, and on the student experience? Can we use the framework to improve instruction?

To do so a framework must be coherent and focused on the right things.

That’s why I start with this big question:

If you had 5 things to focus on in order to build classrooms that produce students who are powerful thinkers, what would they be?

Why 5 (or fewer)?

It’s as many as most people can keep in mind. (In fact, it may be too many to work on at one time.)

If you have 20, you might as well have none. People can’t keep that many things in their heads, and long check lists don’t help. What matters is what people can act on, in teaching and coaching.

What properties should those 5 things have?

They're all you need (there's nothing essential missing).

They each have a certain "integrity" and can be worked on in meaningful ways.

Their framing supports professional growth.

But I didn’t know that was the question to ask when I began the research.

So, I will take you on a brief tour of some years of unsuccessful research.

Will the (Western) literature help?

There are lots of frameworks.

• *Framework for Teaching* (or FFT, developed by Charlotte Danielson of the Danielson Group),

• *Classroom Assessment Scoring System* (or CLASS , developed by Robert Pianta, Karen La Paro, and Bridget Hamre at UVA

• *Protocol for Language Arts Teaching Observations* (or PLATO, developed by Pam Grossman at Stanford University),

• *Mathematical Quality of Instruction* (or MQI, developed by Heather Hill of Harvard University)

• *UTeach Teacher Observation Protocol* (or UTOP, developed by Michael Marder and Candace Walkington at the University of Texas-Austin).

• *Instructional Quality Assessment*, IQA, developed by the University of Pittsburgh.

Actually, No.

They all focus on important things, but they're all partial, or scattered, or have too many random parts; in some way or other none are close enough to use.

They get at different things.

So, we needed to build our own.

Here's our first try, in outline form.

We tried coding lessons, focusing on these things:

	Access (what the teacher gives/allows)	Accountability (what the teacher expects/demands)	Productive Dispositions (what the teacher receives from students)
Strand	Dimensions (codes)	Dimensions (codes)	Dimensions (codes)
Mathematics	Students are able to experience the vibrancy and power of the domain of mathematics	Mathematical exploration and discussion should be accurate. Reasoning and justification should be tied to mathematics.	Students construct mathematics, attempting to discover rather than just receive.
Mathematics Learning	Students are given a chance to learn mathematics. This requires making making mathematics learning practices explicit and accessible.	Students are expected to engage productively in the mathematics learning process, sustain efforts, and contribute to finding solutions.	Students are interested in learning mathematics.
Classroom Community	No students are marginalized in the classroom community. All students have a chance to engage and participate.	Students have an obligation to their teacher and peers to be respectful and helpful. Students are not just participants but leaders of the classroom community.	Students contribute and participate as a community of mathematics practicioners.
Individual Learner	The classroom respects the uniqueness of each individual student, and gives appropriate affordances.	Students have an obligation to themselves to learn mathematics, and productively engage the subject matter.	Students sustain efforts as learners. Students take risks and believe that they can succeed.

It was impossible because of the detail we needed:

Strand	Access (all students have opportunities to engage the subject)		Accountability (students are held to high standards)		Dispositions (student needs are met; students have productive dispositions)		Authority (students have ownership over their engagement with the subject)	
	Dimension	Constructs (codes)	Dimension	Constructs (Codes)	Dimension	Constructs (Codes)	Dimension	Constructs (Codes)
Mathematics	1-1. Access to rich mathematics	a) tasks provide opportunities to engage higher-level mathematical thinking b) the teacher presents tasks in a way that demand rich mathematical engagement	2-1. Accountable to the mathematics	a) teacher and students use multiple representations and make connections between representations; task requires multiple representations and connections between them. b) teacher presses for accuracy c) teacher asks probing questions/elicits reasoning and justification d) teacher and students use academic language e) teacher checks for understanding and provides feedback during instruction f) teacher builds on students' prior knowledge, connects mathematical ideas	3-1. Students view mathematics as:	a) a constructed body of knowledge b) useful	4-1. Authority over mathematical ideas	a) students generate/explain ideas b) students question, challenge, evaluate ideas
Mathematics Learning	1-2. Access to Explicit Expectations (taken from Ball's MQI)	a) teacher is explicit about what students have to do on a given problem b) teacher is explicit about how to use formal math language c) teacher is explicit about how to reason mathematically	2-2 Accountable to mathematics learning	a) teacher expects students to be able to learn mathematics b) teacher expect students to persist in mathematics learning	3-2. Students believe mathematics learning:	a) is achieved through hard work b) requires collaboration c) is rewarding/interesting	4-2. Authority to guide learning processes	a) students facilitate discussions b) students manage logistics c) students set the agenda/have choice in activities
Classroom Community	1-3. Opportunity to Receive (and Give) Meaningful, Constructive Feedback:	a) teacher provides feedback b) students give and receive feedback from other students	2-3. Accountable to classmates	a) discussion among students is math-focused c) teacher relates and connects student ideas to one another d) teacher revoices/marks student contributions e) students question and evaluate each other and teacher	3-3. Dispositions toward classmates	a) students show respect for each others' ideas	4-3. Authority is distributed appropriately throughout the class** **In our scheme, we should be careful to differentiate between normative and non-normative descriptors; it shouldn't look like the ideal is for students to have all the authority and teachers none, or vice versa.	a) across the teacher and the students* b) between pre-existing ideas and ideas generated by the class* *captured by three kinds of "who" in codes cited above: 1) teacher, 2) students, and 3) explicit teacher support for students to engage in X (some codes also imply the additional "who" of outside authorities, such as textbooks or some "They" that might make the rules)
Individual Learner	1-4. Opportunity to Engage the Mathematics in Their Own Way.	a) teacher permits use of non-dominant language b) students engage the mathematics on their own level c) teacher provides students time to work independently d) tasks have multiple entry points e) problem contexts respect students' cultural backgrounds/prior knowledge	2-4. Accountable to themselves	a) students have a role as mathematical authorities b) students sustain efforts to reach learning goals c) students participate in classroom activities	3-4. Students feel:	a) like individuals capable of learning math b) it's okay to make mistakes c) like they have a mathematical future - from Davis & Seashore rubric	4-4. Students acquire authority through competence.	a) teacher positions students as competent b) teacher positions students as "capable" of doing the math - from Ball's MQI and Cohen's complex instruction

Code #	Feasible in Real-time?	Focus Area	Time Scale	Spatial Scale	Description of Code	ACCESS	ACCOUNTABILITY	DISPOSITIONS	AUTHORITY
TEACHER									
1T	Y	Teacher	Lesson	Whole Class	When setting up a task, teacher checks whether students understand the directions	1-a: Explicit Expectations about - what to do on a given task		3-1. Teacher responds to students' disposition toward mathematics as	
2T	Y	Teacher	Lesson	Whole Class	Teacher checks for understanding. (an absolute count of number of times we observe this, either formally through unit assessments, or informally through quick-and-dirty formative in-class quizzes or even small-group questioning)	4-a: Opportunity to Receive Feedback - from the teacher		3-1. Teacher responds to students' disposition toward mathematics as	
3T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher pushes for conceptual understanding (e.g., through "Why?" questions) - (absolute count)	To rich mathematics? (No construct yet about this)	1: Accountability to the Math	3-1. Teacher responds to students' disposition toward mathematics as	4-a: Students positioned as competent (which gives them authority)
4T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher asks students to justify/explain their reasoning. [Is this an example of 3T?]	1-c: Explicit Expectations - about how to reason in math	1: Accountability to the Math		
5T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher prompts students to respond to each other's ideas (absolute count)	4-b: Opportunity to Receive Feedback - from other students	4: Accountability to Classmates	3-2 Teacher responds to students' disposition toward mathematics learning	1-b: Authority to - question, challenge, evaluate math ideas
6T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher solicits student ideas.	3-b: Access to Productive Identities - students positioned as capable learners		3-2 Teacher responds to students' disposition toward mathematics learning	1-a: Authority to - generate/explain math ideas
7T	Y	Teacher	Lesson	Whole Class	Teacher takes up or ignores a student idea. [How does it work to have both "taking up" and "ignoring" as the same code? -NLL]	-		3-2 Teacher responds to students' disposition toward mathematics learning	1-a: Authority to - generate/explain math ideas
8T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher builds on students' prior mathematical knowledge [I need an example here more than on the others; what would this look like? -NLL]	2-b: Engaging the Math in Own Way - on their own math level			
	?	Teacher	Lesson / Unit	Whole Class	Teacher pushes students toward mathematical accuracy and toward formal math terminology [maybe examples would be, "teacher explicitly teaches mathematical language and vocabulary," and/or "teacher revoices student ideas in formal mathematical language." I think is is feasible to code these in real time. -NLL]	1-b: Explicit Expectations about - using formal math terminology	2-1: Accountability to the Math		
	?	Teacher	Unit	-	Teacher makes future-oriented statements about kids using or doing math in the future in some way (Davis & Seashore have a 4-point rubric in their scheme we can look at; We also could make a tally of the number of such statements that occur over the course of a unit; Or, we could just tally yes/no per lesson and then analyze the pattern over the course of the unit)	3-a: Access to Productive Identities - envisioning a mathematical future			
	?	Teacher	Unit	-	Teacher makes an encouraging remark that may, for example, foster persistence or position students as capable learners (We could make a tally of the number of such statements that occur over the course of a unit)	3-b: Access to Productive Identities - students positioned as capable learners		4-a: Students see themselves as capable	1-a: Authority to explain/generate math ideas 4-a: Students positioned as competent (which gives them authority)
	N	Teacher	Lesson	Whole Class / Small Group	Wait time. (calculate the average time a teacher waits for student response after asking a question)	2-c: Engaging the Math in Own Way - students have independent work/think time			
STUDENTS									
4S	Y	Students	Lesson	Whole Class / Small Group	Students justify/explain their reasoning.	-	1: Accountability to the Math		1-a: Authority to - generate/explain math ideas
5S	Y	Students	Lesson	Whole Class / Small Group	Students question and evaluate mathematical ideas, whether they come from the teacher or from classmates. (an absolute count - this may happen in whole group discussion or small-group work)	4-b: Opportunity to Receive Feedback - from other students			1-b: Authority to - question, challenge, evaluate math ideas
6S	Y	Students	Lesson	Whole Class / Small Group	Students share new ideas.	-		3-3. Students dispositions toward classroom community (classmates or the teacher)	1-a: Authority to - generate/explain math ideas
9S	Y	Students	Unit	Whole Class / Small Group	Students facilitate whole-class or small group discussions (yes/no)	-		3-3. Students dispositions toward classroom community (classmates or the teacher)	2-a: Authority over classroom activity - facilitating discussions
10S	Y	Students	Lesson / Unit	Whole Class	Students are responsible for logistical tasks (e.g., passing out papers) - (yes/no)	-		3-4. Students dispositions toward individual/self-efficacy	2-b: Authority over classroom activity - managing logistics
11S	Y	Students	Lesson	Whole Class / Small Group	Especially in classes with ELL students, students are observed using non-dominant language in class without sanction from teacher (yes/no).	2-a: Engaging the Math in Own Way - through use of non-dominant language		3-3. Students dispositions toward classroom community (classmates or the teacher)	4-a: Students positioned as competent (which gives them authority)
12S	Y	Students	Lesson	Whole Class / Small Group	Participation is distributed fairly across students so that no handful of students dominate discussion	2-c: Engaging the Math in Own Way - students have independent work/think time		3-3. Students dispositions toward classroom community (classmates or the teacher)	
	N	Students	Lesson	-	% of time students spend working on math independently (compared with time spent on teacher talking about math or classroom management)	2-c: Engaging the Math in Own Way - students have independent work/think time		3-4. Students dispositions toward individual/self-efficacy	
	?	Students	Lesson / Unit	Whole Class	Students participate in setting lesson agenda and structuring activities (e.g., who to work with, how much time spent on an activity etc.) - (yes/no)	-		3-1. Teacher responds to students' disposition toward mathematics as	2-c: Authority over classroom activity - setting lesson agenda
TASK									
4K	Y	Task	Lesson	-	Task requires students to justify, conjecture, interpret		1: Accountability to the Math	3-1: Nature of mathematics	
	N	Task	Lesson	-	Task affords multiple entry points for students.	2-b: Engaging the Math in Own Way - on their own math level		3-1: Nature of mathematics	4-4b: Students are positioned as competent/capable
	N	Task	Lesson	-	Task affords multiple representations	2-b: Engaging the Math in Own Way - on their own math level	1: Accountability to the Math	3-1: Nature of mathematics	
	?	Task	Lesson	-	Tasks have real-world applications			3-1: Nature of mathematics	

There were codes
For teacher, students,
And task along
All the dimensions.
It was unworkable.

We tried again, simplifying by looking at "Events of Interest." That got complex very fast…

Events of Interest			
Part 3: CAT-specific Events	Sub-Category	Event #	Description of Event
A. Navigating Language		1	Participants rephrase/reword the problem context to put it in more kid-friendly language.
		2	Teacher checks that students understand non-mathematical vocabulary.
		3	Teacher checks that students understand mathematical vocabulary.
		4	Evie: use of reading strategies, students being asked to read aloud or in small groups, word walls, use of personal dictionaries, sentence frames, sentence starters
B. Identifying Relevant Quantities		5	Teacher asks questions that call students attention to relevant quantities (e.g., What is the problem asking you to find? or What does the problem give you?)
		6	Evie: Students connect quantities, operations, relationships, and calculations to reasoning around context.
		7	Evie: Students make sense of the quantities required to solve the problem.
		8	Evie: Students articulate goals or strategies for solving problem connected to reasoning around context.
C. Representing Relevant Quantities	C-1. Articulating Mathematical Relationships Between Quantities	9	Participants make explicit connections between inputs and outputs (vs. relying on recursive rules).
		10	Participants engage in qualitative sense-making of relationships between quantities.
		11	Participants reference a family/families of functions and their features.
	C-2. Generating Representations	12	Kim: Students choose which representation to use
		13	Kim/Dan: Students construct a representation (e.g., equation, graph, table).
		14	Bob: Teacher asks the students to construct a representation / The task requires students to construct a representation.
		15	Alan: The representation is tied in a meaningful or useful way to the context of the problem.
	C-3. Interpreting or Making Connections Between Representations	16	Participants move between representations.
		17	Participants use representations to solve contextual problems.
		18	Participants compare the advantages and/or limitations of various representations.
		19	Evie: participants make connections among representations (it's not just comparing representations, like "I like the table better than a graph"; it's about seeing how the rate of change, for example, shows up in the table and in the graph)
D. Solving the Problem	D-1. Making Calculations or Executing Procedures	20	Bob: Teacher emphasizes arithmetical accuracy or providing opportunities for students to do calculations correctly (providing resources, etc.)
		21	Participants solve an equation for a variable.
		22	Participants use algebraic techniques to solve systems of equations (substitution, elimination, etc. vs. guess-and-check)
	D-2. Attending to the Problem Context to Check the Plausibility of Results or Making Sense of Quantities	23	Participants orally reference the problem context in explaining their work Or Participants reference the problem context in explaining their work in writing.
E. Justifying and Explaining Reasoning		24	?????
		25	?????

So we abandoned that approach as well.

Every approach we took resulted in our looking at a large amount of detail.

We listed hundreds of things that were important to notice. We incorporated everything from the literature and our observations…

ACTION 4.1 2011-11-3

#	Facet				
A	Giving Directions (for Individual or Group Work)	*Setting Process Expectations*	* Setting Product Expectations*		
		1 Teacher tells students to get started without setting process expectations. 2 Teacher sets process expectations (e.g., amount of time for task, how students should organize themselves). 3 Teacher engages students in mutually setting process expectations.	1 Teacher tells students to get started without setting product expectations. 2 Teacher sets expectations about final product (e.g., by providing a scoring rubric, showing examples of high quality work). 3 Teacher engages students in mutually setting expectations for final product.		
B	Summarizing the Math Discussed	Who is Doing the Summarizing?	What is the Nature of the Math Being Summarized?		
		1 2 3	1 2 3		
C	Connecting to Prior Knowledge	Who is Involved in Creating the Connections to Prior Knowledge?	What is the Nature of the Math Being Connected?		
		1 2 3	1 2 3		
D	Positioning Students Relative to Task	Who is Being Positioned as Capable of Doing the Math?	How/Why is the Math Being Learned Relevant/Useful?	What Does it Take to Be Successful in Math?	
		1 Teacher tells students to work on task but doesn't position them relative to the task. 2 Teacher positions students as capable of working on a difficult task, but addresses students in a general way (e.g., you guys can do this). 3 Teacher is explicit in positioning ALL students as capable of working on the task (e.g., multiple ability treatment).	1 Mathematics is not emphasized as important/relevant to students. 2 Teacher talks about the importance of mathematics for students in a general sense (e.g., you guys really need to know this). 3 Utility of math is addressed specifically (e.g. students are positioned as having mathematical futures).	1 Teacher doesn't emphasize effort over ability. 2 Teacher emphasizes the importance of effort. 3 Teacher emphasizes the importance of effort AND the need to be persistent in the face of difficulty.	
E	Teacher Exposition of Mathematical Ideas	[Incorporating Ideas from Class Discussion into Exposition]	[Depth/Quality of the Math in the Exposition]		
		1 Teacher ignores or dismisses student reasoning. 2 Teacher acknowledges contribution but doesn't actively incorporate it into the lesson (e.g., that's an interesting idea, but we're not working on that now). 3 Teacher incorporates and builds on student reasoning to move the lesson forward	1 2 3		
F	Discussing Mathematical Ideas/Reasoning	[Facilitating Discussion Participants]	[Eliciting Student Reasoning]	[How Student Responses are Taken Up]	[Encouraging Multiple Solution Paths]
		1 Only the first student that raises his/her hand is the one that gets called on. 2 Beyond the first student, at least one other student who raised his/her hand gets called on to respond to a given question. 3 Teacher uses techniques to actively engage students who do not volunteer (e.g., wait time, popsicle sticks, cold calling).	1 Teacher does not attempt to further explicate student's thinking. 2 Teacher attempts to explain/re-phrase the students' thinking. 3 Teacher probes student to further explicate his/her strategy/thinking.	1 2 3	1 The task/introduction strongly suggests a single solution path. 2 The task/introduction affords multiple potential solution paths. 3 The task/introduction encourages/requires multiple solution paths and/or the contrast of different solutions.
G	Monitoring Whole Class Understanding - INFORMAL	How Deep was the Math Being Assessed?	How Many Students are We Getting Data From?	What Does the Teacher Do with This Information?	
		1 The monitoring only involved checking answers (i.e., "How many of you got 3/4 for #17?") 2 The monitoring had to do with assessing students' execution of a mathematical procedure. 3 The monitoring asked students to explain their reasoning or answer a <u>why</u> question.	1 2 3	1 2 3	
H	Monitoring Whole Class Understanding - FORMAL	How Deep was the Math Being Assessed?	How Many Students are We Getting Data From?		
		1 The monitoring only involved checking answers (i.e., "How many of you got 3/4 for #17?") 2 The monitoring had to do with assessing students' execution of a mathematical procedure. 3 The monitoring asked students to explain their reasoning or answer a <u>why</u> question.	1 2 3		
I	Student Seeks to Clarify Mathematical Ideas/Reveals Confusion	How Cognitively Demanding is the Response?	How Cognitively Demanding is the Student's Question?	How is the Question Taken Up?	
		1 Teacher ignores or dismisses the question. 2 Teacher gives an explanation directly answering the student's question. 3 Teacher engages the student/class in answering the question (e.g., acting as a guide).	1 The student asks about whether an answer is correct or not (i.e., a "WHAT" question) or a non-specific question (e.g., "I don't know how to get started!") 2 The student asks a specific question about HOW to do a procedure. 3 The student asks WHY something works.	1 Acknowledged but not responded to. 2 Answered by teacher. 3 Students engaged in answering it.	
J	Scaffolding the Mathematics in the Tasks	[Maintaining Cognitive Demand]	[Providing a Variety of Entry Points]		
		1 2 3	1 2 3		

Here's a closer Look…

#	Facet			
A	**Giving Directions (for Individual or Group Work)**	***Setting Process Expectations***	*** Setting Product Expectations***	
		1 Teacher tells students to get started without setting process expectations. 2 Teacher sets process expectations (e.g., amount of time for task, how students should organize themselves). 3 Teacher engages students in mutually setting process expectations.	1 Teacher tells students to get started without setting product expectations. 2 Teacher sets expectations about final product (e.g., by providing a scoring rubric, showing examples of high quality work). 3 Teacher engages students in mutually setting expectations for final product.	
B	**Summarizing the Math Discussed**	**Who is Doing the Summarizing?**	**What is the Nature of the Math Being Summarized?**	
		1 2 3	1 2 3	
C	**Connecting to Prior Knowledge**	**Who is Involved in Creating the Connections to Prior Knowledge?**	**What is the Nature of the Math Being Connected?**	
		1 2 3	1 2 3	
D	**Positioning Students Relative to Task**	**Who is Being Positioned as Capable of Doing the Math?**	**How/Why is the Math Being Learned Relevant/Useful?**	**What Does it Take to Be Successful in Math?**
		1 Teacher tells students to work on task but doesn't position them relative to the task. 2 Teacher positions students as capable of working on a difficult task, but addresses students in a general way (e.g., you guys can do this). 3 Teacher is explicit in positioning ALL students as capable of working on the task (e.g., multiple ability treatment).	1 Mathematics is not emphasized as important/relevant to students. 2 Teacher talks about the importance of mathematics for students in a general sense (e.g., you guys really need to know this). 3 Utility of math is addressed specifically (e.g. students are positioned as having mathematical futures).	1 Teacher doesn't emphasize effort over ability. 2 Teacher emphasizes the importance of effort. 3 Teacher emphasizes the importance of effort AND the need to be persistent in the face of difficulty.

And then I realized…

Why not create equivalence classes, clustering all of these “things to look at” into meaningful categories?
Here is the result…

The Five Dimensions of Powerful Mathematics Classrooms

The Mathematics	Cognitive Demand	Equitable Access to Mathematics	Agency, Ownership, and Identity	Formative Assessment
The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful mathematical thinkers. Discussions are focused and coherent, providing opportunities to learn mathematical ideas, techniques, and perspectives, make connections, and develop productive mathematical habits of mind.	*The extent to which students have opportunities to grapple with and make sense of important mathematical ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematical content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.*	*The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about mathematical ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Note how this framework focuses on the student point of view.

Four of the five dimensions have to do with the ways in which the students experience the mathematics.

What's essential about this framework?

Here are 5 central points.

Five central points about TRU:

1. **The TRU Dimensions are necessary and sufficient. That is,**

   If things go well along all 5 dimensions, students will emerge from the classroom as powerful thinkers.
   If things go badly along *any* of the dimensions, they will not.

Five central points about TRU:

2. **TRU involves a fundamental shift in perspective, from teacher-centered to student-centered.**

The key question is *not:*
"Do I like what the teacher is doing?"
It is:
"What does instruction feel like, from the point of view of the student?"

Observe the Lesson Through a Student's Eyes

The Content	• What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Content	• Do I get to participate in meaningful math learning? • Can I hide or be ignored? In what ways am I kept engaged?
Agency, Ownership, and Identity	• What opportunities do I have to explain my ideas? In what ways are they built on? • How am I recognized as being capable and able to contribute?
Formative Assessment	• How is my thinking included in classroom discussions? • Does instruction respond to my ideas and help me think more deeply?

Five central points about TRU:

3. **TRU does not tell you how to teach, because there are many different ways to be an effective teacher.**

TRU serves to *problematize* instruction. That is: Asking, "how am I doing along this dimension; how can I improve?" can lead to richer instruction without imposing a particular style or norms on teachers.

Five central points about TRU:

4. **TRU is NOT a tool or set of tools.** TRU is a perspective regarding what counts in instruction, and TRU provides a language for talking about instruction in powerful ways. With this understanding, you can make use of any productive tools wisely.

But we have tools, of course…

See

http://TRUFramework.org

Five central points about TRU:

5. TRU doesn't compete with other initiatives; it works with them and makes them stronger.

You can use it to "problematize" the approaches you take.

The challenge(s), if you think TRU might be a useful frame:

How does one go about validating it (in my own Western context)?
How does one build an R&D agenda?
How does one compare and contrast internationally?

Validation, Part 1:

While creating the framework, look at videos of teachers known to be effective. Do they do well on the emerging framework?

Validation, Part 2:

Show people videos and see what they comment on. Are their comments consistent with the categories in the framework?

Validation, Part 3:

Create a scoring rubric. Use a database that has classroom videos as well as classroom scores on tests of mathematical thinking and problem solving. See if scores on the rubric correlate with scores on the math tests.

Building an R&D Agenda, 1

Create tools and make them widely available.

See https://truframework.org/

and

http://map.mathshell.org/...

TEACHING FOR ROBUST UNDERSTANDING FRAMEWORK

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FALS support mathematical sense-making

What is the TRU framework?

TRU is a framework for characterizing powerful learning environments in crisp and map.mathshell.org

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Mathematics Assessment Project

ASSESSING 21ST CENTURY MATH

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The Mathematics Assessment Project is part of the Math Design Collaborative initiated by the Bill & Melinda Gates Foundation. The project set out to design and develop well-engineered tools for formative and summative assessment that expose students' mathematical knowledge and reasoning, helping teachers guide them towards improvement and monitor progress. The tools are relevant to any curriculum that seeks to deepen students' understanding of mathematical concepts and develop their ability to apply that knowledge to non-routine problems.

▸ More about the Math Assessment Project

▶ Lessons

Formative Assessment Lessons: *Classroom Challenges*

100 lessons for formative assessment, some focused on developing math concepts, others on solving non-routine problems. A Brief Guide for teachers and administrators (PDF) is recommended reading before using these lessons for the first time.

▶ Tasks

Summative Assessment Tasks

A set of 94 exemplar summative assessment tasks to illustrate the range of performance goals required by CCSSM. The tasks come with scoring rubrics and examples of scored student work.

▶ Tests

Prototype Tests

Complete summative test forms and rubrics designed to help teachers and students monitor their progress using a range of task types similar to the 'Tasks' section.

▶ PD Modules

Professional Development Modules

5 Prototype modules that encourage groups of teachers to explore the practical and pedagogical concepts behind the materials, such as formative assessment, collaborative learning and the use of unstructured problems.

▶ TRU Math Suite

The TRU Math Tools Suite

The Teaching for Robust Understanding of Mathematics (TRU Math) suite is a set of tools with applications in Professional Development and research based around a framework for characterizing powerful learning environments.

Tools for School and District Leaders

NEW

The MathNIC project has released free tools to help schools and school districts be more effective in organizing for improvement, supporting teaching and learning, and communicating with parents and the community. Visit mathnic.org for details.

ICMI Awards

Hugh Burkhardt and Malcolm Swan have received a prestigious award from ICMI for the team's work in Math Education.
Read more…

RFA/CRESST Report

The *Classroom Challenges* are central to Research for Action's report on the MDC's Influence on Teaching and Learning.

Free to Schools

All our materials can be downloaded for free and may be reproduced as-is for non-commercial use. Precise terms vary between materials. Enquiries to: map.info@mathshell.org.

State, district and CCSSI standards appear courtesy of their respective authors.

A Tool for Planning for and Reflecting on Teaching

The ***TRU Conversation Guide*** is designed to foster reflective conversations about instruction.

Frame each dimension with questions:

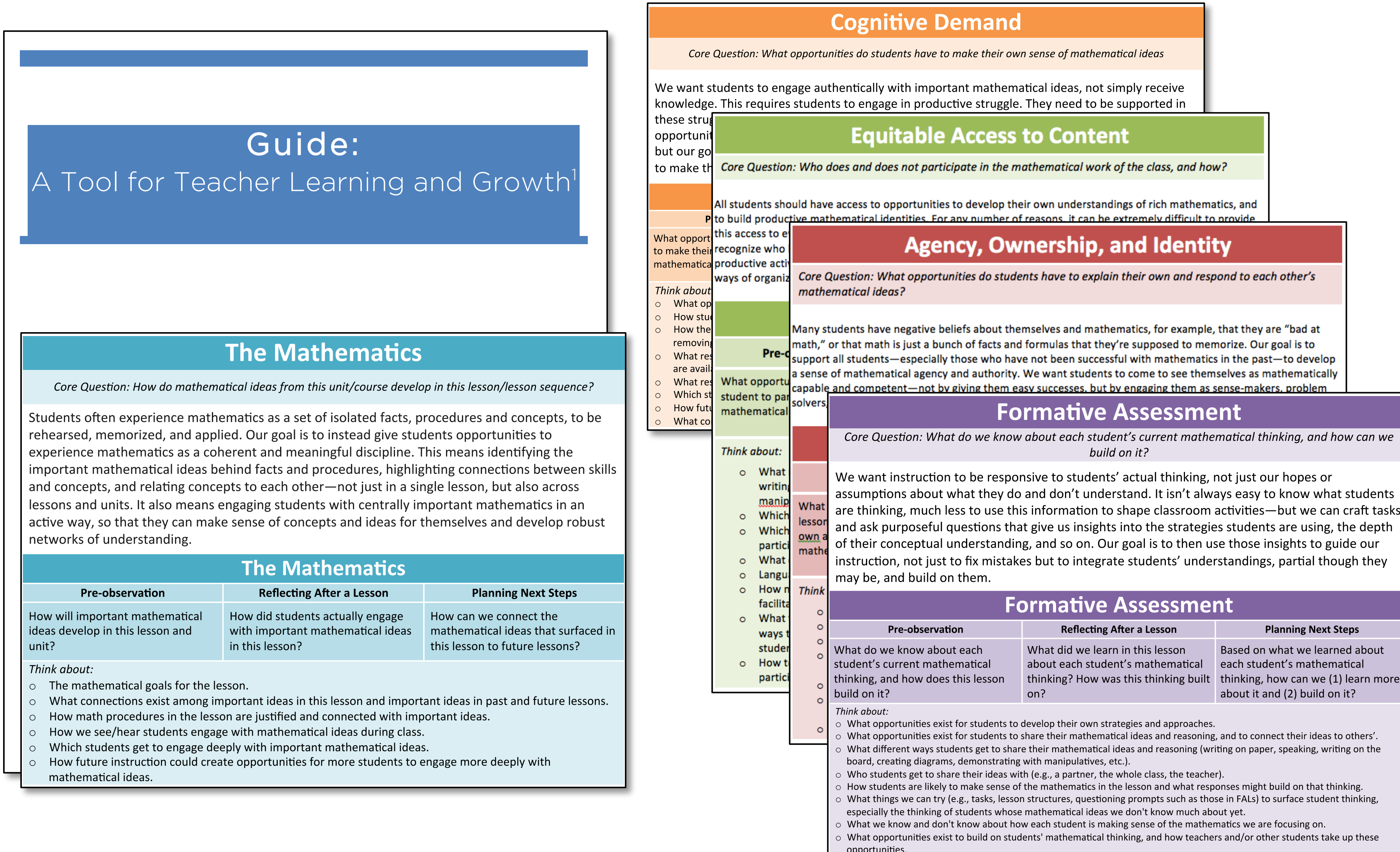
The Content
How do ideas from this unit/course develop in this lesson/lesson sequence?
Cognitive Demand
What opportunities do students have to make their own sense of important ideas?
Equitable Access to Content
Who does and does not participate in the meaningful work of the class, and how?
Agency, Ownership, and Identity
What opportunities do students have to explain their own and respond to each other's ideas?
Formative Assessment
What do we know about each student's current thinking, and how can we build on it?

. . . and expand the questions,

to *problematize* instruction.

That is: Ask a series of questions that help to plan for instruction that provides students with deeper opportunities along each of the five dimensions.

The TRU Conversation Guide



Guide:
A Tool for Teacher Learning and Growth[1]

Cognitive Demand

Core Question: What opportunities do students have to make their own sense of mathematical ideas

We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This requires students to engage in productive struggle. They need to be supported in

Equitable Access to Content

Core Question: Who does and does not participate in the mathematical work of the class, and how?

All students should have access to opportunities to develop their own understandings of rich mathematics, and

Agency, Ownership, and Identity

Core Question: What opportunities do students have to explain their own and respond to each other's mathematical ideas?

Many students have negative beliefs about themselves and mathematics, for example, that they are "bad at math," or that math is just a bunch of facts and formulas that they're supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and authority. We want students to come to see themselves as mathematically

The Mathematics

Core Question: How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?

Students often experience mathematics as a set of isolated facts, procedures and concepts, to be rehearsed, memorized, and applied. Our goal is to instead give students opportunities to experience mathematics as a coherent and meaningful discipline. This means identifying the important mathematical ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It also means engaging students with centrally important mathematics in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding.

The Mathematics

Pre-observation	Reflecting After a Lesson	Planning Next Steps
How will important mathematical ideas develop in this lesson and unit?	How did students actually engage with important mathematical ideas in this lesson?	How can we connect the mathematical ideas that surfaced in this lesson to future lessons?

Think about:

- The mathematical goals for the lesson.
- What connections exist among important ideas in this lesson and important ideas in past and future lessons.
- How math procedures in the lesson are justified and connected with important ideas.
- How we see/hear students engage with mathematical ideas during class.
- Which students get to engage deeply with important mathematical ideas.
- How future instruction could create opportunities for more students to engage more deeply with mathematical ideas.

Formative Assessment

Core Question: What do we know about each student's current mathematical thinking, and how can we build on it?

We want instruction to be responsive to students' actual thinking, not just our hopes or assumptions about what they do and don't understand. It isn't always easy to know what students are thinking, much less to use this information to shape classroom activities—but we can craft tasks and ask purposeful questions that give us insights into the strategies students are using, the depth of their conceptual understanding, and so on. Our goal is to then use those insights to guide our instruction, not just to fix mistakes but to integrate students' understandings, partial though they may be, and build on them.

Formative Assessment

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What do we know about each student's current mathematical thinking, and how does this lesson build on it?	What did we learn in this lesson about each student's mathematical thinking? How was this thinking built on?	Based on what we learned about each student's mathematical thinking, how can we (1) learn more about it and (2) build on it?

Think about:

- What opportunities exist for students to develop their own strategies and approaches.
- What opportunities exist for students to share their mathematical ideas and reasoning, and to connect their ideas to others'.
- What different ways students get to share their mathematical ideas and reasoning (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with manipulatives, etc.).
- Who students get to share their ideas with (e.g., a partner, the whole class, the teacher).
- How students are likely to make sense of the mathematics in the lesson and what responses might build on that thinking.
- What things we can try (e.g., tasks, lesson structures, questioning prompts such as those in FALs) to surface student thinking, especially the thinking of students whose mathematical ideas we don't know much about yet.
- What we know and don't know about how each student is making sense of the mathematics we are focusing on.
- What opportunities exist to build on students' mathematical thinking, and how teachers and/or other students take up these opportunities.

Agency, Ownership, and Identity

Core Questions: What opportunities do students have to see themselves and each other as powerful doers of mathematics? How can we create more of these opportunities?

Many students have negative beliefs about themselves and mathematics, for example, that they are "bad at math," or that math is just a bunch of facts and formulas that they're supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and ownership over their own learning. We want students to come to see themselves as mathematically capable and competent—not by giving them easy successes, but by engaging them as sense-makers, problem solvers, and creators of mathematical ideas.

Planning

What opportunities might exist for students to generate and explain their own ideas? To respond to each other's ideas? How can we create more opportunities?

Reflecting

How have we seen students explain their own and respond to each other's ideas? What has that looked and sounded like in specific cases?

Things to think about

- Who generates the ideas that get discussed?
- What kinds of ideas do students have opportunities to generate and share (strategies, connections, partial understandings, prior knowledge, representations)?
- Who evaluates and/or responds to others' ideas?
- How deeply do students get to explain their ideas?
- How does (or how could) the teacher respond to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.)?
- How are norms about students' and teachers' roles in generating ideas developing?
- How are norms about what counts as mathematical activity (justifying, experimenting, connecting, practicing, memorizing, etc.) developing?
- Which students get to explain their own ideas? To respond to others' ideas in meaningful ways?
- Which students seem to see themselves as powerful mathematical thinkers right now?
- How might we create more opportunities for more students to see themselves and each other as powerful mathematical thinkers?

To support collegial observations,

we offer the

TRU Observation Guide,

Which highlights things to look for is a lesson is going well.

The guide can be used by coaches or TLCs for planning and debriefing classroom observations…

The TRU Observation Guide

The TRU Observation Guide:

A Tool for Teachers, Coaches, and Professional Learning Communities

THE CONTENT

The extent to which central disciplinary ideas and methods, as represented by State or National Standards, are present and embodied in instruction. Every student should have opportunities to grapple meaningfully with important ideas and to develop productive disciplinary habits of mind. Teachers should have opportunities to consider and discuss how each lesson's objective connects to the big ideas and practices they want students to develop over time.

Each Student...	Teachers...
• Engages with grade level content in ways that highlight important information, concepts, and methods • Has opportunities to develop productive disciplinary habits of mind • Has opportunities to reason about disciplinary issues, both orally and in writing, using appropriate academic language • Explains their reasoning processes as well as their answers.	• Highlight important ideas and provide opportunities for students to engage with them • Use materials or assignments that center on key ideas, connections, and applications • Explicitly connect the lesson's big ideas to what has come before and will be done in the future • Support the purposeful use of academic language and other representations central to the discipline • Support students in seeing the discipline as being coherent, connected, and comprehensible

• Other focal points for observation:

What are the big ideas in this lesson? How do they connect to what has come before, and/or establish a base for future work? How do the ways students engage with the material support the development of conceptual understanding and the development of disciplinary habits of mind?

Goal: All students work on core disciplinary issues in ways that enable them to develop conceptual understandings, develop reasoning and problem solving skills, and use disciplinary concepts, tools and methods in relevant contexts.

COGNITIVE DEMAND

The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to every student's deepening understanding of disciplinary content

EQUITABLE ACCESS TO CONTENT

The extent to which classroom activities invite and support the meaningful engagement with core content by all students. Finding ways to support the diverse range of learners in engaging

AGENCY, OWNERSHIP, AND IDENTITY

The extent to which every student has opportunities to explore, conjecture, reason, explain, and build on emerging ideas, contributing to the development of agency (the willingness to engage

FORMATIVE ASSESSMENT

The extent to which classroom activities elicit all students' thinking and subsequent interactions respond to that thinking, by building on productive beginnings or by addressing emerging misunderstandings. High quality instruction "meets students where they are" and gives them opportunities to develop deeper understandings, both as shaped by the teacher and in student-to-student interactions.

Each student...	Teachers...
• Explains their thinking, even if somewhat preliminary • Sees errors as opportunities for new learning • Consistently reflects on their work and the work of peers • Sees fellow students as resources for their own learning • Provides specific and accurate feedback to fellow students • Makes use of feedback in revising their work	• Create safe climates in which students feel free to express their ideas and understandings • Use materials that elicit multiple strategies, and have students explain their reasoning, in order to gain information about student' emerging understandings • Flexibly adjust content and process, providing students opportunities for re-engagement and revision • Provide timely and specific feedback to students, as part of classroom routines that prompt students to make active use of feedback to further their learning • Create opportunities for students' individual and collaborative reflection on their knowledge and learning

• Other focal points for observation:

What opportunities exist for all students to demonstrate their understandings? What opportunities exist to build on the thinking that is revealed? How do teachers and/or other students take up these opportunities? Where can more be created?

Goal: Every student's learning is continually enhanced by the ongoing strategic and flexible use of techniques and activities that allow students to reveal their emerging understandings, and that provide opportunities both to rethink misunderstandings to build on productive ideas.

AGENCY, OWNERSHIP, AND IDENTITY

The extent to which every student has opportunities to explore, conjecture, reason, explain, and build on emerging ideas, contributing to the development of agency (the willingness to engage academically) and ownership over the content, resulting in positive mathematical identities.

Each student…	Teachers…
• Takes ownership of the learning process in planning, monitoring, and reflecting on individual and/or collective work • Asks questions and makes suggestions that support analyzing, evaluating, applying and synthesizing mathematical ideas • Builds on the contributions of others and help others see or make connections • Holds classmates and themselves accountable for justifying their positions, through the use of evidence and/or elaborating on their reasoning	• Provide time for students to develop and express mathematical ideas and reasoning • Work to make sure all students have opportunities to have their voices heard • Encourage student-to-student discussions and promote productive exchanges • Assign tasks and pose questions that call for mathematical justification, and for students to explain their reasoning • Employ a range of techniques that attribute ideas to students, to build student ownership and identity

• Other focal points for observation:

What opportunities do all students have to see themselves and others as proficient mathematical thinkers, to grapple with challenges and construct new understandings, to build on others' ideas, and demonstrate their understandings? How can more of these opportunities be created?

Goal: All students build productive mathematical identities through taking advantage of opportunities to engage meaningfully with the discipline and share and refine their developing ideas.

The first version of the Observation Guide was actually built by San Francisco Unified School District, and it's being used in a number of school districts across the US.

So, these ideas work at the "ground level." They're not just "academic."

Building an R&D Agenda, 2

As suggested above, make your tools widely available so other researchers can use them. Collaborate with school districts to get "real world data."

Collaborations:

TRU is used in New York, Chicago, and San Francisco. Many of our partners are building tools, adding to the work. Colleagues in China, England, France, Germany, Japan, Israel, and Singapore are also working with the ideas.

Building an R&D Agenda, 3

Look into mechanism.
* What kinds of teacher learning communities can we support?
* How do we document changes in teacher understanding? In teachers' practices?
* In student behavior, as well as student learning?

International Comparisons

I am particularly interested in how these ideas do or do not make sense in China. From what I know, there are some systematic differences in cultural context:

Comparisons with China, Issue 1

In China there is more of a focus on the teaching and the lesson, less on the students (compare Chinese and Japanese Lesson Study, for example). Does TRU, which is student-focused, seem too strange?

Comparisons with China, Issue 2

TRU Dimension 4, “Agency, Ownership, and Identity” is a very Western idea. Does it make sense in the Chinese context?

Comparisons with China to come…

I am collaborating with Yu-Liang Chang (張宇樑) from Taiwan. It will be good to see what directions our collaborations take, and if they can spread!

Thank you!

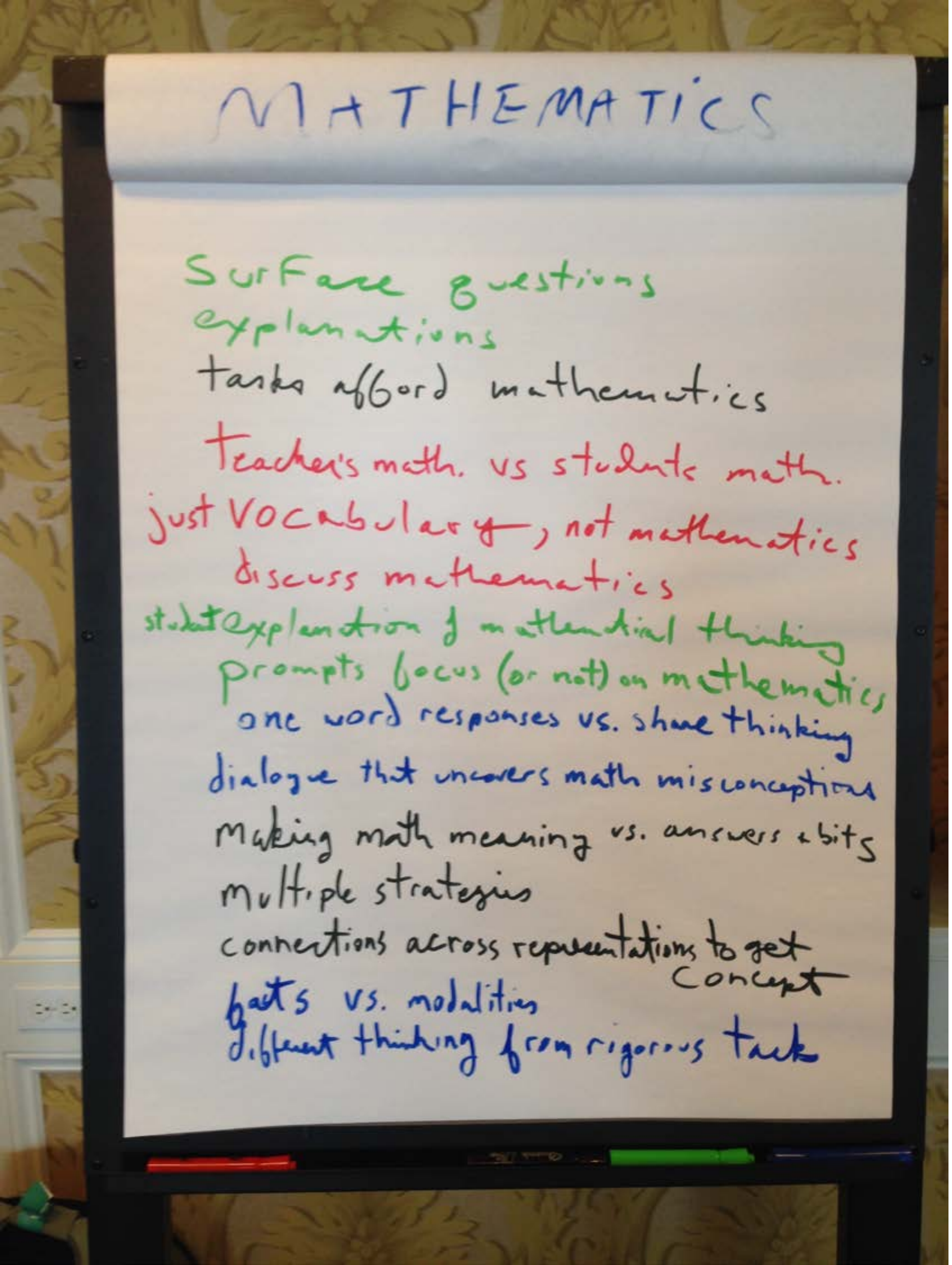
謝謝

Extras, part 1:
What happens when people look at classroom videos.

Every time a group looks at videos, there are lots of comments about what the teachers are doing, and what it must feel like to be a student in their classrooms.

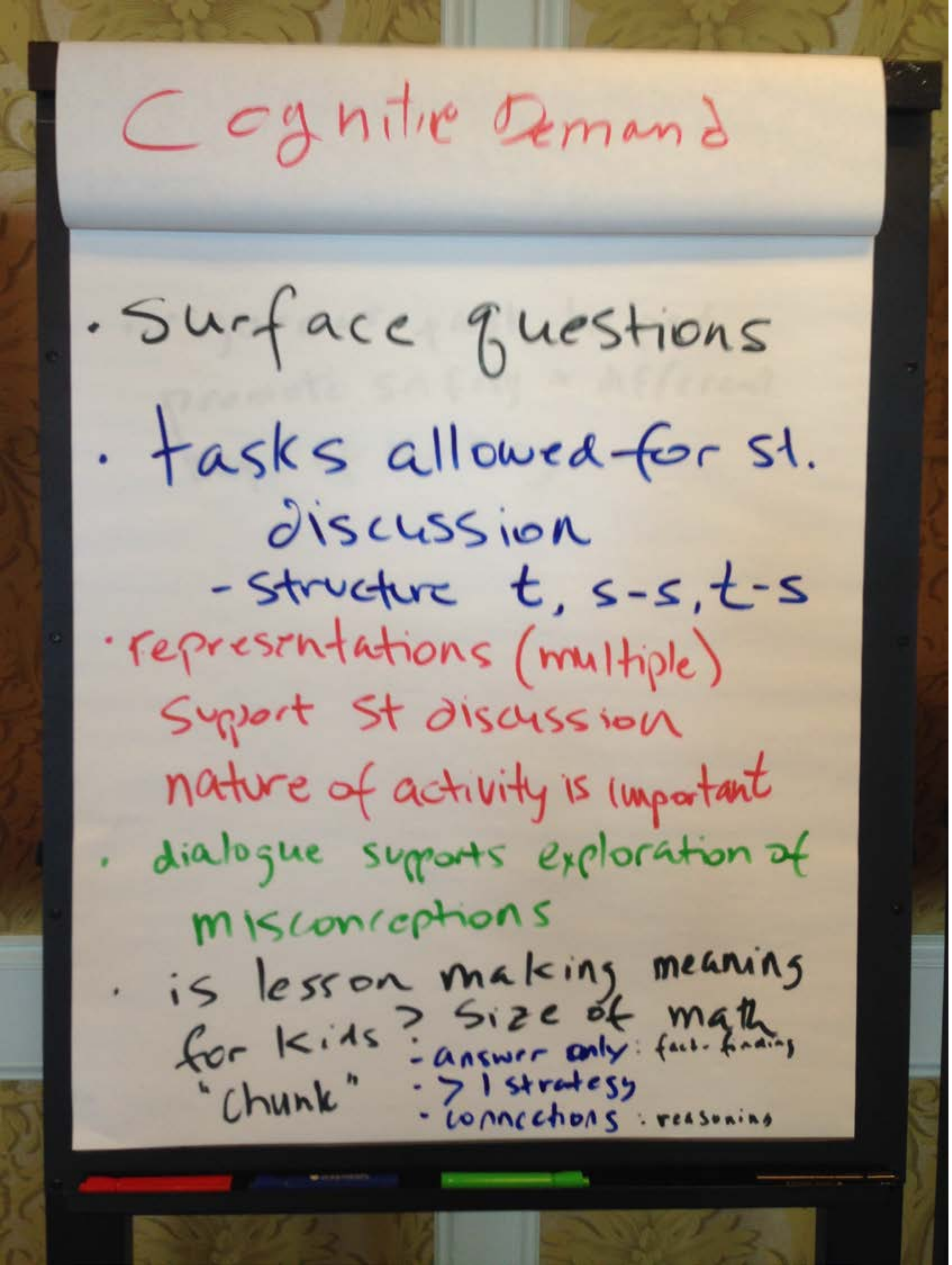
And every time, it is easy to organize everything they say into five categories.

Let's see what we've got…



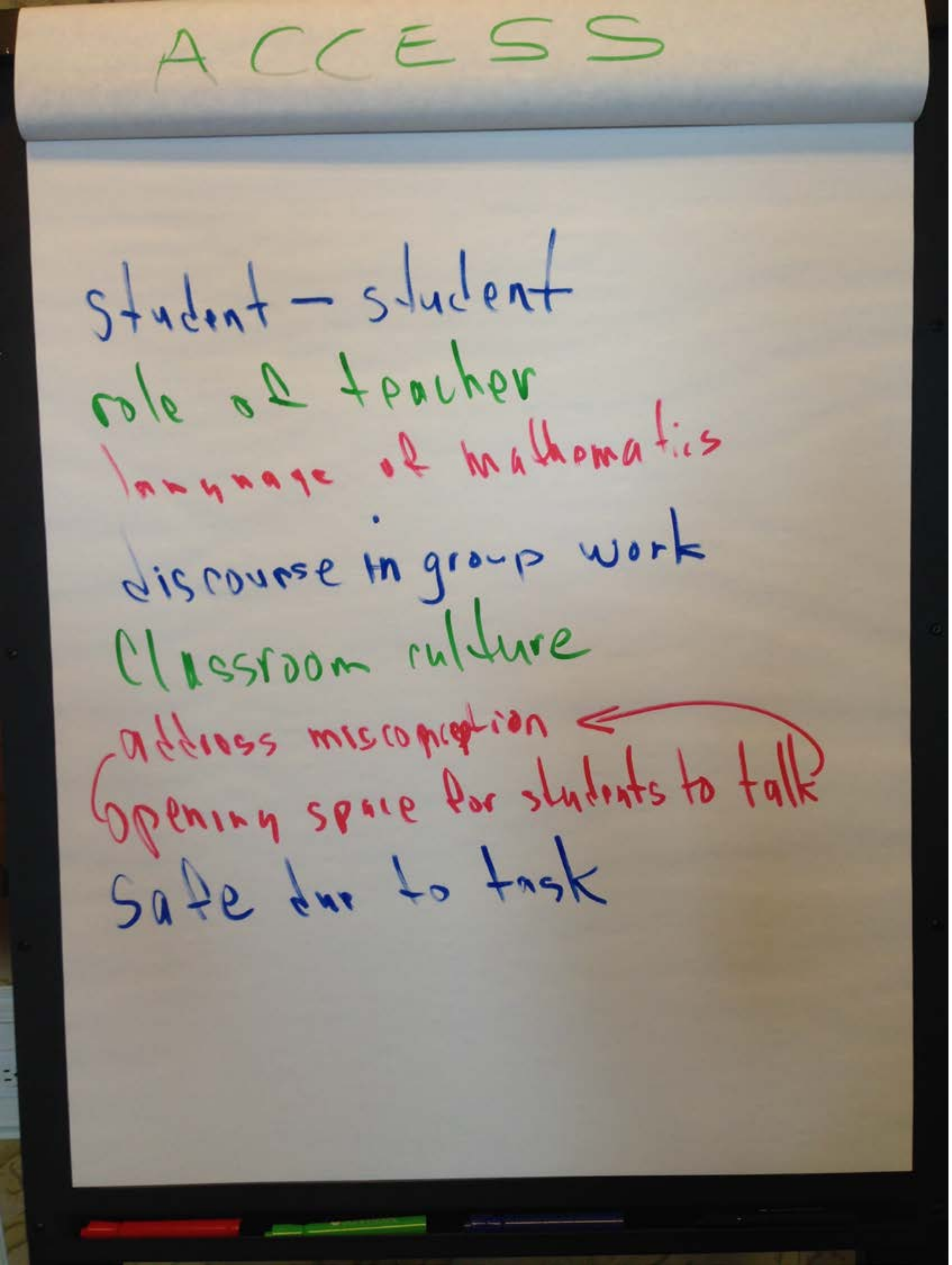
The Mathematics

Is it important, coherent, connected? Where are the big ideas? Are there opportunities for thinking and problem solving?



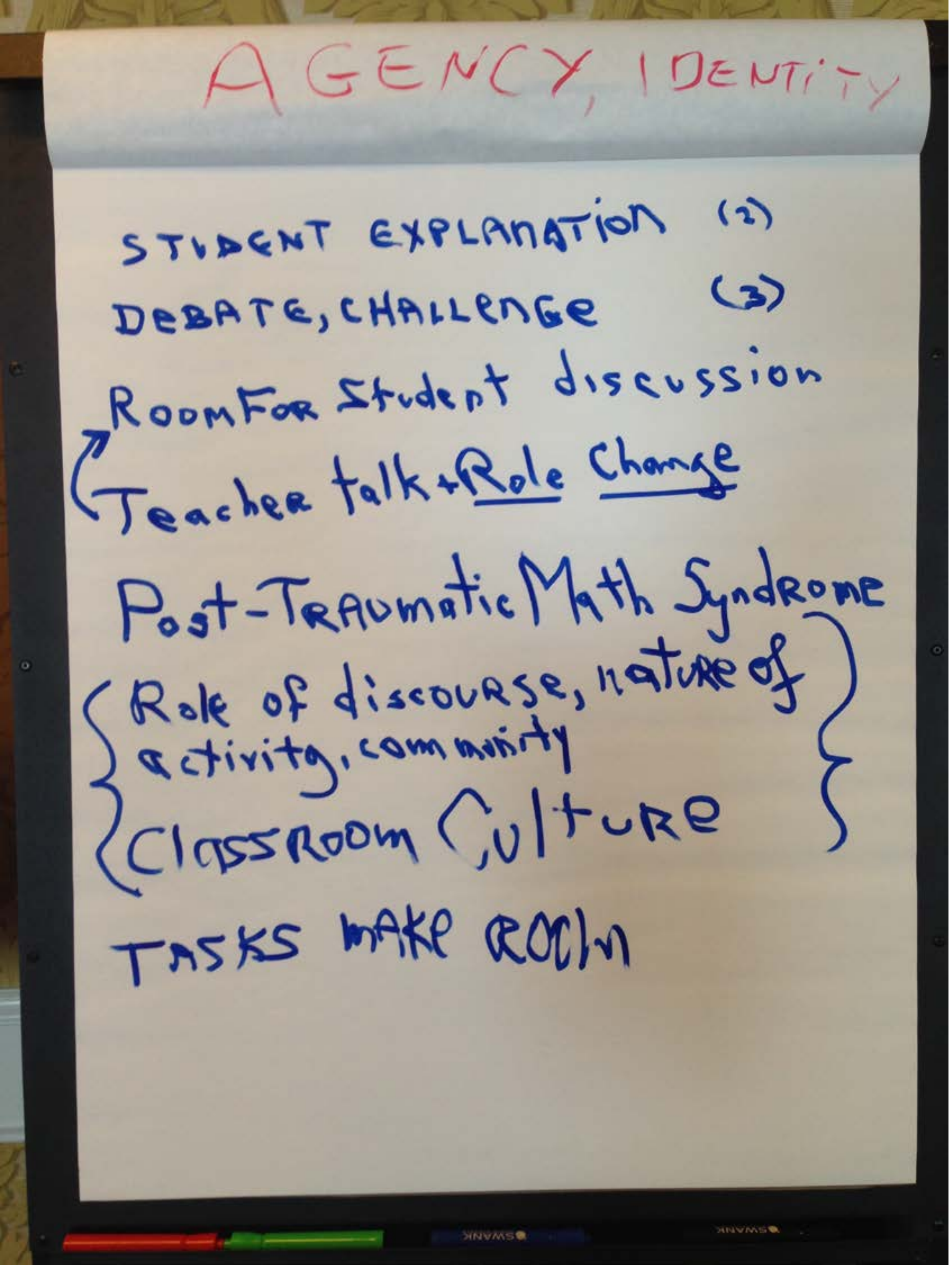
Cognitive Demand

Do the students have opportunities for sense making – for “productive struggle,” engaging productively with the mathematics?



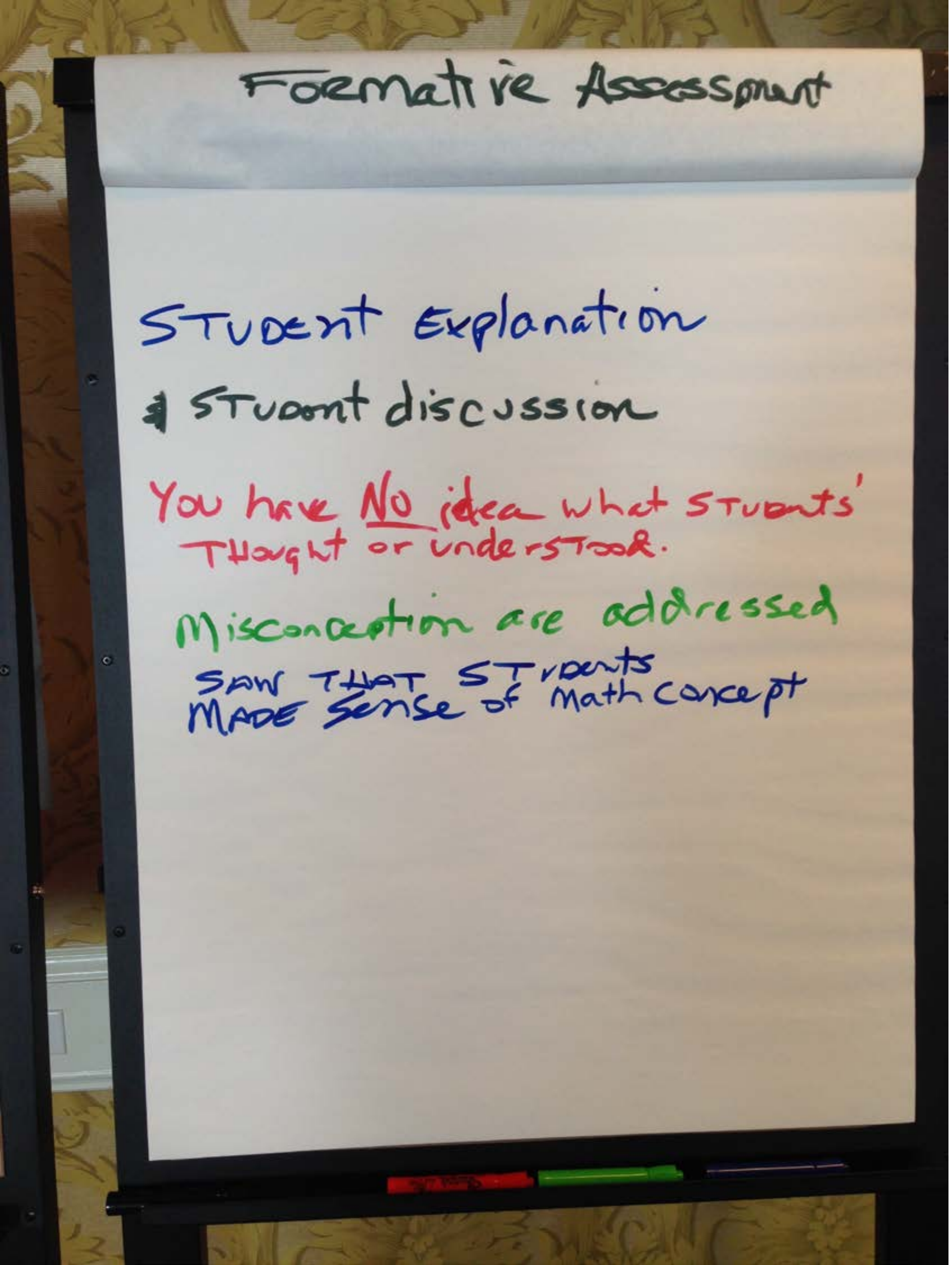
Access and Equity

Who participates, in what ways? Are there opportunities for *every* student to engage in sense making?



Agency and Identity

Do students have the opportunities to do and talk mathematics? Do they come to see themselves as "math people," or people who cannot do mathematics?

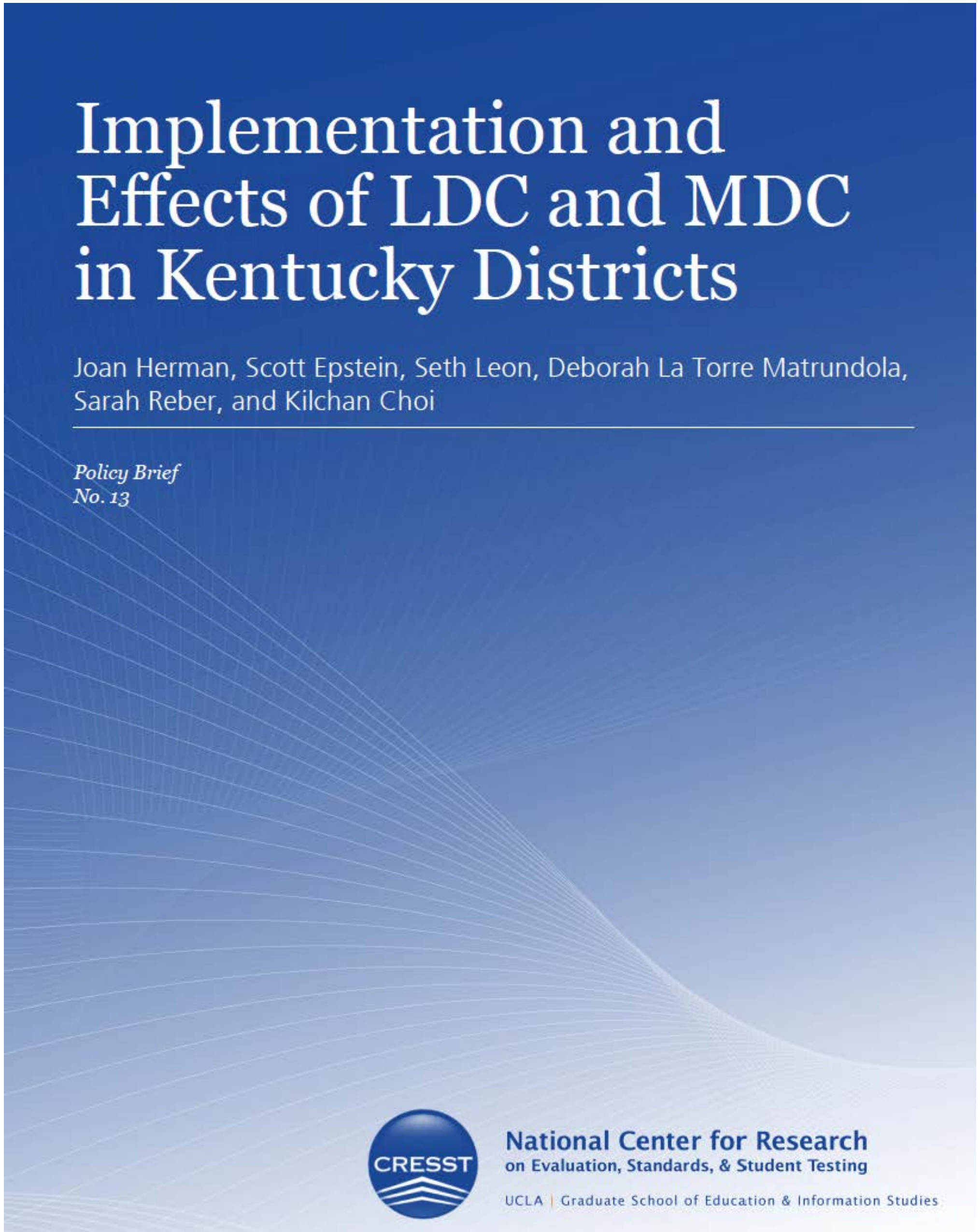


Formative Assessment

Does classroom discussion reveal what students understand, so that instruction can be adjusted for purposes of helping students learn?

Do these tools make a difference?

Here are some data.



Implementation and Effects of LDC and MDC in Kentucky Districts

Joan Herman, Scott Epstein, Seth Leon, Deborah La Torre Matrundola, Sarah Reber, and Kilchan Choi

Policy Brief
No. 13

CRESST

National Center for Research on Evaluation, Standards, & Student Testing

UCLA | Graduate School of Education & Information Studies

MDC = "Math design Collaborative," which was designed to help implement the Formative Assessment Lessons.

The results:

Participating teachers were expected to implement between four and six Formative Assessment Lessons, meaning that students were engaged only 8-12 days of the school year.

Nonetheless, the studies found statistically significant learning effects of approximately **4.6 months** for the Formative Assessment Lessons.

Why?

The teachers learn TRU-related techniques that they use in their regular instruction – our desired "multiplier effect."

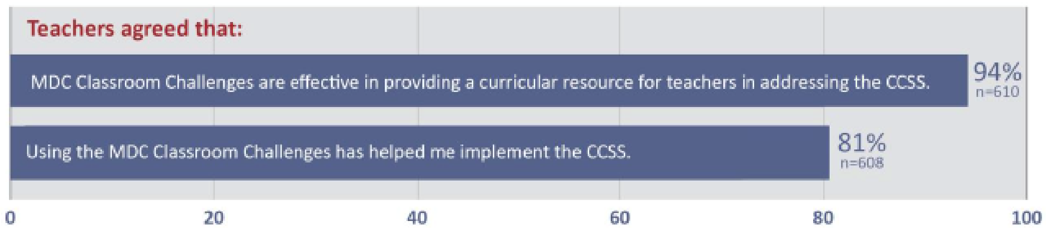
Here's a recent study:

MDC's Influence on Teaching and Learning

Prepared by Research for Action

February 2015

1. MDC Helps Implement the Core Common State Standards

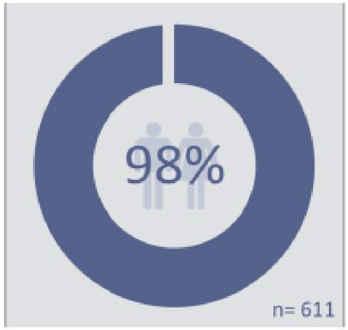


The vast majority of teachers reported that the Classroom Challenges were supporting their implementation of the Core Common State Standards. As one high school math teacher said:

> *I do [think participating in MDC will help me teach the Common Core]. Because the common core is all about "do fewer things better." We want to be able to get into these investigative things, and we want to be able to emphasize reasoning over mechanics, and so this is dead on that.*

2. MDC Supports Teaching as Coaching

Almost all participating teachers indicated that the role of teacher as instructional "facilitator" or "coach," which is embodied in the Challenges, supports increasing students' mathematical understanding. Compared to providing direct instruction, coaching enables students to take on a more active learning role.

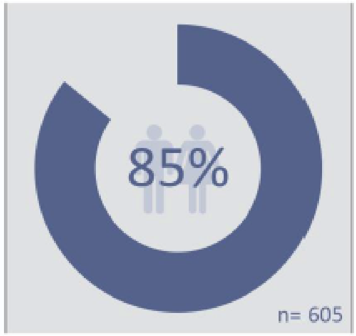


Teachers agreed that the teacher taking on the role of facilitator coach strengthens students' mathematical understanding.

> *I've been teaching for 36 years, and teaching the same way. It's hard to change; to teach an old dog new tricks. But now that I'm doing it, I love it....At first, I felt like, I'm not teaching! [laughs] But now I realize that they really are learning, and doing more on their own. And I don't have to stand up there, and teach my heart out, and they [are] just looking at me and still not getting it. Now...they're probably learning more. – High school math teacher*

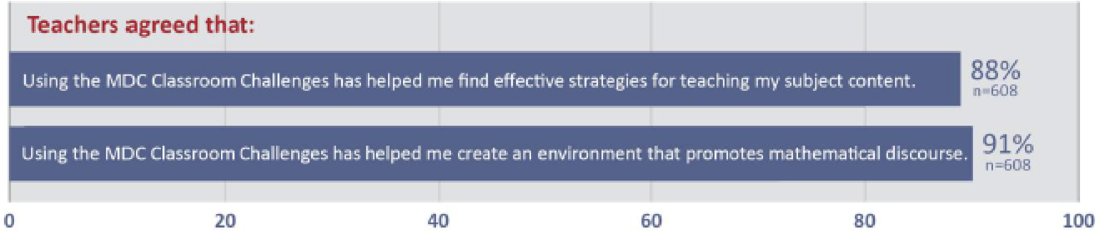
3. MDC is Raising Teachers' Expectations

The Classroom Challenges are rooted in the rigorous demands of the CCSS and designed to raise the level of mathematical content in instruction. Teachers reported that the Classroom Challenges were increasing their academic expectations for their students.



Teacher respondents agreed that using the MDC Classroom Challenges raised their expectations for students' mathematical work.

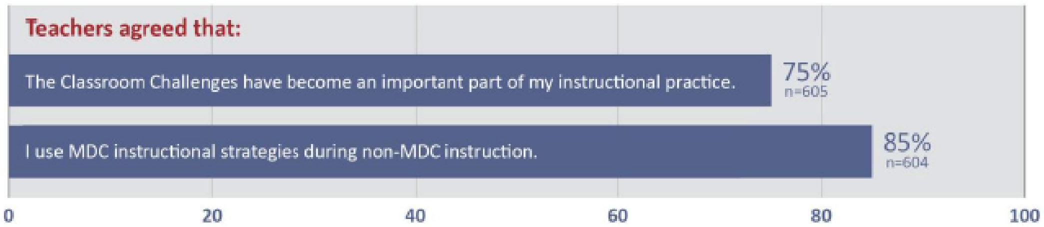
4. MDC Provides Effective Teaching Strategies and Changes Overall Instruction



The vast majority of MDC teachers reported that the lessons provided them with effective strategies for teaching math and strengthening mathematical discourse in their classrooms.

> *The students actually talk about math and they are actually having debates and they are debating between who is correct. Before, without this type of teaching, they never talked about math. It was always the teacher talking and they never got into good discussions or justify their answers, and they were never responsible for understanding what other people were thinking as well. – High school math teacher*

In addition, teachers reported that MDC practices were affecting their instruction, even when they weren't using the Challenges.

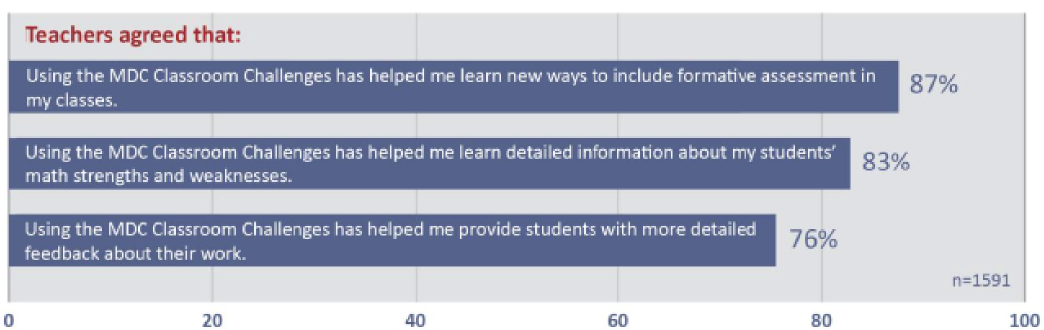


At least three-quarters of MDC teachers said that the lessons had become important to their instructional practice and that they were infusing strategies from the Classroom Challenges into their ongoing instruction.

High school math teachers reported:

> *This has expanded me to do more work in groups, even more than I have done in the past.*
>
> *I think it's helping us grow as teachers in how we question the students.*
>
> *It has definitely made me more aware of putting the responsibility on them-for them to be their own learners and I love the questioning technique and being their facilitator to learning. It has definitely changed my way of teaching.*

5. MDC Offers Formative Assessment



Large majorities of MDC teachers agreed that using the lessons helped them incorporate more formative assessment in their classes, learn information about students' math strengths and weaknesses and give students more detailed feedback about their work. In interviews, teachers reported that analyzing the pre-assessment and post-assessment enabled them to identify gaps in student knowledge and detect growth. Teachers also reported using information about students' misconceptions to develop feedback questions or re-teach content.